

Solutions

Q.1 (a)

$$\text{Let } P(x) = 6x^4 - 2x^2 + 7x + 10$$

Since $1 - 2x$ divides $P(x)$

By remainder theorem,

$$\begin{aligned}\text{Remainder} &= P\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^4 - 2\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) + 10 \\ &= \frac{6}{16} - \frac{2}{4} + \frac{7}{2} + 10 \\ &= \frac{107}{8}\end{aligned}$$

Q.2 (b)

$$\text{Let } f(x) = Kx^2 + (K - 2)x + 4$$

$$\text{Given, } f(1) = 0$$

$$K(1)^2 + (K - 2)(1) + 4 = 0$$

$$\therefore K + K - 2 + 4 = 0$$

$$\therefore 2K = -2$$

$$\therefore K = -1$$

Q.3 (d)

For $4x^2 + 1$, all the exponents of x are non-negative integers.

Q.4 (b)

$$x^2 + bx + c = (x - \alpha)(x - \beta)$$

$$x^2 + bx + c = x^2 - (\alpha + \beta)x + \alpha\beta$$

By comparing

$$\alpha + \beta = -b$$

$$\alpha\beta = c$$

$$\alpha + \beta + \alpha\beta = -b + c = c - b$$

Q.5 (a)

$$\text{Use } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Q.6 (c)

$$x - \frac{1}{x} = 1, \text{ square it}$$

$$\left(x - \frac{1}{x}\right)^2 = 1^2, \quad x^2 + \frac{1}{x^2} - 2 = 1$$

$$x^2 + \frac{1}{x^2} = 3$$

Q.7 (b)

$$x + 4y = 14 \quad \times 7$$

$$7x - 3y = 5 \quad \times 1$$

$$7x + 28y = 98$$

$$-7x + 3y = -5$$

$$\hline 31y = 93$$

$$y = 3$$

$$x = 14 - 4y = 14 - 4 \times (3) = 14 - 12$$

$$x = 2$$

$$(x, y) = (2, 3)$$

Q.8 (a)

Theoretical

Q.9 (d)

$$\frac{12}{x} + \frac{3}{y} = 3$$

$$\frac{12}{6} + \frac{3}{y} = 3$$

$$2 + \frac{3}{y} = 3$$

$$\frac{3}{y} = 1$$

$$\therefore y = 3$$

Q.10 (d)

$$x^2 - 2x - 35 = 0$$

$$(x - 7)(x + 5) = 0$$

$$\therefore x = 7 \text{ or } x = -5$$

Sum of square of roots

$$= (7)^2 + (-5)^2$$

$$= 49 + 25$$

$$= 74$$

Q.11 (b)

$$2x^2 - 3x + 1 = 0$$

$$a = 2, b = -3, c = 1$$

$$D = b^2 - 4ac$$

$$= (-3)^2 - 4 \times 2 \times 1 = 9 - 8 = 1$$

Q.12 (c)

$$2^x + 2^{1-x} = 3$$

$$2^x + \frac{2}{2^x} = 3$$

$$\text{Let } t = 2^x$$

$$t + \frac{2}{t} = 3$$

$$t^2 - 3t + 2 = 0$$

$$(t-2)(t-1) = 0$$

$$\therefore t = 2 \text{ \& } t = 1$$

$$\therefore 2^x = 2 \text{ \& } 2^x = 1$$

$$\therefore x = 1 \text{ \& } x = 0$$

Q.13 (a)

$$t_4 = -1$$

$$\therefore a + 3d = -1 \quad \dots(1)$$

$$t_9 = -16$$

$$\therefore a + 8d = -16 \quad \dots(2)$$

On solving (1) & (2), we get

$$d = -3, a = 8$$

Q.14 (b)

$$t_3 = \frac{1}{12}$$

$$\therefore ar^2 = \frac{1}{12} \quad \dots(1)$$

$$t_6 = -\frac{1}{96}$$

$$\therefore ar^5 = -\frac{1}{96} \quad \dots(2)$$

On solving (1) & (2), we get

$$r = -\frac{1}{2}, a = \frac{1}{3}$$

Q.15 (c)

p, q, r are in A.P.

$$\therefore 2q = p + r$$

x, y, z are in G.P.

$$\therefore \frac{y}{x} = \frac{z}{y}$$

$$x^{q-r} y^{r-p} z^{p-q}$$

$$= \frac{x^q}{x^r} \cdot \frac{y^r}{y^p} \cdot \frac{z^p}{z^q}$$

$$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^r \left(\frac{z}{y}\right)^p$$

$$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^r \left(\frac{y}{x}\right)^p \quad \dots \left(\because \frac{z}{y} = \frac{y}{x}\right)$$

$$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^{p+r}$$

$$= \left(\frac{x}{z}\right)^q \left(\frac{y}{x}\right)^{2q} \quad \dots \left(\because 2q = p + r\right)$$

$$= \frac{x^q}{z^q} \cdot \frac{y^{2q}}{x^{2q}}$$

$$= \frac{(y^2)^q}{(xz)^q}$$

$$= \frac{(y^2)^q}{(y^2)^q} \quad \dots \left(\because y^2 = xz\right)$$

$$= 1$$

Q.16 (d)

$$2(k+7) = (2k+3) + (3k-4) \quad \dots (\text{Property of A.P.})$$

$$\therefore 2k + 14 = 2k + 3 + 3k - 4$$

$$\therefore 3k = 15$$

$$\therefore k = 5$$

Q.17 (a)

$$a = 7, r = 2$$

$$S_n = a \left(\frac{r^n - 1}{r - 1} \right)$$

$$S_9 = 7 \left(\frac{2^9 - 1}{2 - 1} \right)$$

$$= 7(512 - 1)$$

$$= 7 \times 511$$

$$\therefore S_9 = 3577$$

Q.18 (b)

$$|3x-5| = \frac{17}{2}$$

$$3x-5 = \pm \frac{17}{2}$$

$$3x-5 = \frac{17}{2}$$

$$3x-5 = -\frac{17}{2}$$

$$3x = \frac{17}{2} + 5$$

$$3x = -\frac{17}{2} + 5$$

$$3x = \frac{27}{2}$$

$$3x = -\frac{7}{2}$$

$$x = \frac{9}{2}$$

$$x = -\frac{7}{6}$$

$$\text{Sum of all } x = \frac{9}{2} - \frac{7}{6} = \frac{27}{6} - \frac{7}{6} = \frac{20}{6} = \frac{10}{3}$$

Q.19 (a)

Theoretical

Q.20 (b)

$$f(x) = 3x^2 - 5x + 6k$$

Let α be one zero of $f(x)$.

Then $\frac{1}{\alpha}$ is other zero of $f(x)$.

By Vieta's formula

$$\alpha \times \frac{1}{\alpha} = \frac{6k}{3}$$

$$1 = 2k$$

$$k = \frac{1}{2}$$

Q.21 (b)

Let α and β are roots of required quadratic equation

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{Given: } \frac{\alpha + \beta}{2} = 15 \text{ and } \sqrt{\alpha\beta} = 12$$

$$\therefore \alpha + \beta = 30 \text{ and } \alpha\beta = 144$$

$$\therefore x^2 - 30x + 144 = 0$$

Q.22 (a)

$$\text{Let } t = \sqrt{30 + \sqrt{30 + \sqrt{30 + \dots \infty}}}$$

$$t = \sqrt{30 + t}$$

$$t^2 = 30 + t$$

$$(t-6)(t+5) = 0$$

$$t = 6 \text{ and } t = -5$$

Since square root is always positive.

$$\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots \infty}}} = 6$$

Q.23 (d)

Let x students planned for a picnic and per person food cost is y .

$$\therefore xy = 400$$

$$\therefore (x-10)(y+20) = 400$$

$$\therefore xy = xy + 20x - 10y - 200$$

$$\therefore 20x - 10y = 200$$

$$\therefore 2x - y = 20$$

$$\therefore y = 2x - 20$$

$$\therefore x(2x - 20) = 400 \quad \dots (\because xy = 400)$$

$$\therefore 2x^2 - 20x - 400 = 0$$

$$\therefore x^2 - 10x - 200 = 0$$

$$\therefore (x-20)(x+10) = 0$$

$$\therefore x = 20 \text{ and } x = -10$$

Q.24 (a)

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{Given, } S_p = S_q$$

$$\frac{p}{2} [2a + (p-1)d] = \frac{q}{2} [2a + (q-1)d]$$

$$\therefore 2ap + p(p-1)d = 2aq + q(q-1)d$$

$$\therefore 2ap - 2aq = q(q-1)d - p(p-1)d$$

$$\therefore 2a(p-q) = d[q(q-1) - p(p-1)]$$

$$\therefore 2a(p-q) = d[q^2 - q - p^2 + p]$$

$$\therefore 2a(p-q) = d[(q-p)(q+p) - (q-p)]$$

$$\therefore 2a(p-q) = d(q-p)(q+p-1)$$

$$\text{Since } p \neq q, 2a = -d(q+p-1)$$

$$\therefore 2a + d(p+q-1) = 0$$

$$S_{p+q} = \frac{p+q}{2} [2a + (p+q-1)d] = \frac{p+q}{2} \times 0$$

$$\therefore S_{p+q} = 0$$

Q.25 (b)

The principal amount invested for every = 1000

The amount at the end of 100 years for the principal amount, $P = A = P \left(1 + \frac{r}{100}\right)^n$

Where P = Principal amount = 1000/-

r = Rate of interest = 12%

N = Number of years = 10

The amount invested in the first year will earn an interest for 10 years and the amount in the second year will earn an interest for 9 years and so on. Let us make a chart to find the amounts at the end of 10th year.

Year	Amount Invested	Interest calculated for Years	Amount of the end of 10 th year
1 st	1000	10	$1000 \left(1 + \frac{12}{100}\right)^{10} = \left(\frac{112}{100}\right)^{10} \times 10^3$
2 nd	1000	9	$1000 \left(1 + \frac{12}{100}\right)^9 = \left(\frac{112}{100}\right)^9 \times 10^3$
3 rd	1000	8	$1000 \left(1 + \frac{12}{100}\right)^8 = \left(\frac{112}{100}\right)^8 \times 10^3$
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The earning of the man for every years investment at the end

of 10th year are $10^3 \left(\frac{112}{100}\right)^{10}, 10^3 \left(\frac{112}{100}\right)^9, \dots$

The sum of all these earnings will give the total amount earned by him after 10 years

$= 10^3 \left(\frac{112}{100}\right)^{10} + 10^9 + 10^3 \times \left(\frac{112}{100}\right)^8 + \dots$

The above series forms a geometric progression with first term $= a = 10^3 \left(\frac{112}{100}\right)$

Common ratio $= r = \left(\frac{100}{112}\right)$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\therefore S_{10} = \frac{10^3 \left(\frac{112}{100}\right)^{10} \left[\left(\frac{100}{112}\right)^{10} - 1 \right]}{\left(\frac{100}{112} - 1\right)} = 19652.5 \text{ Rs. (approx)}$$

The total amount saved by the man at the rate of 12% for Rs. 1000 per year for 10 years is 10,000 for which he gets a return of Rs. 19,652.5

Q.26 (b)

XYZ is a three-digit number.

So, we can write this as : $100X + 10Y + Z$

$$XYZ + YZX + ZXY$$

$$= 100X + 10Y + Z + 100Y + 10Z + X + 100Z + 10X + Y$$

$$= 111X + 111Y + 111Z$$

$$= 111(X + Y + Z)$$

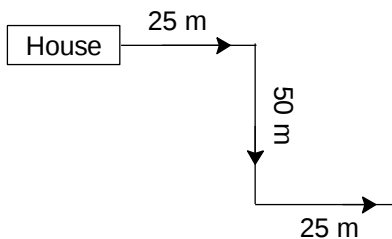
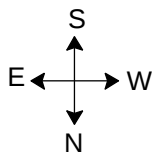
Hence, $XYZ + YZX + ZXY$ is divisible by $X + Y + Z$ and 111.

Also 111 is divisible by 3 and 37.

Since $X + Y + Z$ is not divisible by 3.

$XYZ + YZX + ZXY$ is not divisible by 9.

Q.27 (d)



By looking at the diagram we can easily say that the direction of man from the starting point is North-West.

Q.28 (a)

2, 6, 12, 20, 30, X, 56

$$2 = 2^2 - 2$$

$$6 = 3^2 - 3$$

$$12 = 4^2 - 4$$

$$20 = 5^2 - 5$$

$$30 = 6^2 - 6$$

$$X = 7^2 - 7 = 49 - 7 = 42$$

$$56 = 8^2 - 8$$

Q.29 (b)

$$\otimes + 1\otimes + 5\otimes + \otimes\otimes + \otimes 1 = 1\otimes\otimes$$

Let $\otimes = x$ where $x \in \{1, 2, \dots, 9\}$

$$x + 1x + 5x + xx + x1 = 1xx$$

$$x + (10 + x) + (50 + x) + (10x + x) + (10x + 1) = 100 + 10x + x$$

$$24x + 61 = 100 + 11x$$

$$13x = 39$$

$$x = 3$$

$$\otimes = 3$$

G H I	
7 8 9	$789 \times 2 = 1578$
D E F	
4 5 6	$456 \times 2 = 912$
A B C	
1 2 3	$123 \times 2 = 246$